

A new twist to preheating

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Metric perturbations typically strengthen field resonances during preheating. In contrast we present a model in which the super-Hubble field resonances are completely *suppressed* when metric perturbations are included. The model is the nonminimal Fakir-Unruh scenario which is exactly solvable in the long-wavelength limit when metric perturbations are included, but exhibits exponential growth of super-Hubble modes in their absence. This gravitationally enhanced integrability is exceptional, both for its rarity and for the power with which it illustrates the importance of including metric perturbations in consistent studies of preheating. We conjecture a no-go result - there exists no *single-field* model with growth of cosmologically-relevant metric perturbations during preheating.

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I. INTRODUCTION

Gravity has persistently proven the most difficult of the forces of Nature to bring under theoretical control, both at the classical and quantum levels. This is due, in part, to the extreme non-linear and mixed hyperbolic-elliptic nature of the field equations. Further, the dimensional gravitational coupling constant, $G = m_{\text{pl}}^{-2}$, leads to strong non-renormalizability of even the linearized, quantum, theory in four dimensions.

The semi-classical approximation was developed in partial response to these obstacles. While there are strong indications that large regions of the semi-classical solution space may be spurious [1], a consistent cosmology of inflation in the post-Planckian universe has emerged. This appears to lie in the semi-classical domain with gravity playing a gentle, domesticated, rôle of quiet order within a highly symmetric universe.

Preheating, in contrast, is a violent, non-equilibrium and anarchic epoch [2,3] which may have linked this Utopian regime to the older, radiation-dominated, universe. The rapid transfer of energy between fields during preheating can excite the tiny metric perturbations produced during inflation, even on cosmological scales and may lead to unbridled growth and non-linearity [4]. This threatens to take the system away from the safety of the semi-classical approximation [5].

In these multi-field cases, neglecting metric perturbations can be a grave mistake. Nevertheless, a belief has developed that metric perturbations are unimportant for understanding the super-Hubble evolution of a single scalar field [6]. This is true in the minimally coupled case [7], but certainly *not* true in the *non-minimally* coupled case we discuss below.

Since the non-minimal case was the only single field model in which super-Hubble resonances might plausibly have existed, we are lead to conjecture that there exist no single-field models with super-Hubble metric resonances.

It was pointed out by Fakir and Unruh [8] that the fine-

tuning of λ , which plagues the minimal $V(\phi) = \lambda\phi^4/4$ chaotic inflation model, is absent when ξ is very large and negative (see also [9,10]). Recently Tsujikawa *et al.* [11] showed that long-wave modes of the inflaton in this model are resonantly enhanced during preheating for $\xi \ll -1$, *when metric perturbations are neglected*, due to the oscillations of the Ricci scalar [12]. Naively one might expect this geometric preheating to cause super-Hubble modes of the gauge-invariant metric perturbations to grow too [5]. In fact, we will show that including metric perturbations has a more surprising effect, namely to completely *remove* the super-Hubble resonances.

II. THE MODEL AND ANALYTICAL ESTIMATES

Consider the inflaton, ϕ , coupled non-minimally to the spacetime curvature R with Lagrangian density:

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) - \frac{1}{2} \xi R\phi^2 \right], \quad (2.1)$$

with $G \equiv \kappa^2/8\pi = m_{\text{pl}}^{-2}$ the gravitational coupling constant, and ξ the non-minimal coupling. For a massive inflaton, $|\xi| \lesssim 10^{-3}$ is required for sufficient inflation. On the other hand, in the massless case with self-interaction

$$V(\phi) = \lambda\phi^4/4, \quad (2.2)$$

such a constraint is absent for negative ξ . In this paper, we consider the Fakir-Unruh scenario [8] where the potential is described by (2.2) with negative ξ .*

We choose a flat FLRW background and consider the perturbed metric in the longitudinal gauge:

*We consider negative ξ since large positive ξ leads to a repulsive effective gravitational coupling.

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)\delta_{ij}dx^i dx^j, \quad (2.3)$$

where Φ and Ψ are gauge-invariant potentials [7]. We decompose the inflaton field as $\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x})$, where ϕ_0 is the homogeneous condensate and $\delta\phi$ is the gauge-invariant fluctuation. The Fourier modes of the first order perturbed Einstein equations are then as [13]

$$\Phi_k = \Psi_k - \delta F_k / F \quad (2.4)$$

$$\dot{\Psi}_k + \left(H + \frac{\dot{F}}{F}\right) \Phi_k = \frac{\kappa^2}{2F} \left(\dot{\phi}_0 \delta\phi_k + \delta\dot{F}_k - H\delta F_k\right), \quad (2.5)$$

$$\begin{aligned} & \ddot{\Psi}_k + H\dot{\Psi}_k + \left(H + \frac{\dot{F}}{2F}\right) (2\dot{\Psi}_k + \dot{\Phi}_k) + \frac{\kappa^2 \lambda \phi_0^4}{4F} \Phi_k \\ &= \frac{1}{2F} \left[\delta\ddot{F}_k + 2H\delta\dot{F}_k - \left(\frac{\kappa^2 \lambda \phi_0^4}{4} - \frac{1}{2}\ddot{F} - \frac{5}{2}H\dot{F}_k\right) \frac{\delta F_k}{F} \right. \\ & \left. + \kappa^2 \left\{ \dot{\phi}_0 \delta\dot{\phi}_k - (\xi R + \lambda \phi_0^2) \phi_0 \delta\phi_k \right\} \right], \quad (2.6) \end{aligned}$$

$$\begin{aligned} & \delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + (k^2/a^2 + 3\lambda\phi_0^2 + \xi R) \delta\phi_k \\ &= \dot{\phi}_0(\dot{\Phi}_k + 6H\Phi_k + 3\dot{\Psi}_k) + 2\ddot{\phi}_0\Phi_k + 2\xi\phi_0 \\ & \times \left[3(2\dot{H}\Phi_k + H\dot{\Phi}_k + \ddot{\Psi}_k + 4H^2\Phi_k + 4H\dot{\Psi}_k) \right. \\ & \left. - k^2/a^2(\Phi_k - 2\Psi_k) \right], \quad (2.7) \end{aligned}$$

where $F = 1 - \xi\kappa^2\phi_0^2$, $\delta F_k = -2\xi\kappa^2\phi_0\delta\phi_k$, and $H \equiv \dot{a}/a$ is the Hubble expansion rate. From Eq. (2.4) it is clear that Φ_k and Ψ_k do not coincide in the non-minimally coupled case, due to the non-vanishing anisotropic stress in this case. We include the backreaction of inflaton fluctuations on the background equations for the scale factor and the condensate ϕ_0 , via the Hartree approximation [11,18,17], yielding

$$\begin{aligned} H^2 &= \frac{\kappa^2}{3(1 - \xi\kappa^2\langle\phi^2\rangle)} \left[\frac{1}{2}(\dot{\phi}_0^2 + \langle\delta\dot{\phi}^2\rangle) + \left(\frac{1}{2} - 2\xi\right) \langle\delta\phi'^2\rangle \right. \\ & \left. + \frac{1}{4}\lambda(\phi_0^4 + 6\phi_0^2\langle\delta\phi^2\rangle + 3\langle\delta\phi^2\rangle^2) \right. \\ & \left. + 2\xi\{3H(\phi_0\dot{\phi}_0 + \langle\delta\phi\delta\dot{\phi}\rangle) - \langle\delta\phi\delta\phi''\rangle\} \right], \quad (2.8) \end{aligned}$$

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \lambda\phi_0(\phi_0^2 + 3\langle\delta\phi^2\rangle) + \xi R\phi_0 = 0, \quad (2.9)$$

where R is the scalar curvature, given by

$$\begin{aligned} R &= \frac{\kappa^2}{1 - \xi\kappa^2\langle\phi^2\rangle} \left[(1 - 6\xi)(-\dot{\phi}_0^2 - \langle\delta\dot{\phi}^2\rangle + \langle\delta\phi'^2\rangle) \right. \\ & \left. + \lambda(\phi_0^4 + 6\phi_0^2\langle\delta\phi^2\rangle + 3\langle\delta\phi^2\rangle^2) + 6\xi\{\phi_0\ddot{\phi}_0 \right. \\ & \left. + \langle\delta\phi\delta\ddot{\phi}\rangle + 3H(\phi_0\dot{\phi}_0 + \langle\delta\phi\delta\dot{\phi}\rangle) - \langle\delta\phi\delta\phi''\rangle\} \right]. \quad (2.10) \end{aligned}$$

The expectation values of $\delta\phi^2$ and ϕ^2 are defined as

$$\langle\delta\phi^2\rangle = \frac{1}{2\pi^2} \int k^2 |\delta\phi_k|^2 dk, \quad (2.11)$$

$$\langle\phi^2\rangle = \phi_0^2 + \langle\delta\phi^2\rangle. \quad (2.12)$$

Since the growth of the variance $\langle\delta\phi^2\rangle$ affects the evolution of the fluctuations, we change ϕ_0^2 in Eqs. (2.4)-(2.7) to $\langle\phi^2\rangle$. In principle we should also consider the backreaction due to growth of metric perturbations [14]. However, we shall see that this is unnecessary here since long-wavelength modes do not grow resonantly. Further, the Hartree approximation misses rescattering effects [3], which may be important at the final stage of preheating. We leave both of these issues for future work.

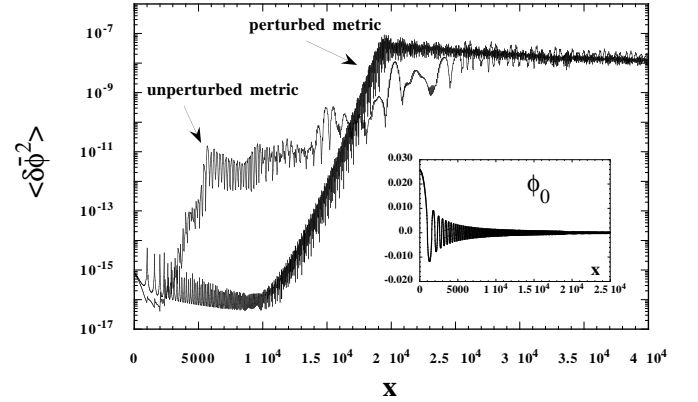


FIG. 1: Time evolution of the inflaton variance $\langle\delta\phi^2\rangle \equiv \langle\delta\phi^2\rangle/m_{\text{pl}}^2$ for $\xi = -70$ and $\lambda = 10^{-12}$, both for the perturbed and unperturbed ($\Phi_k = 0$) metrics. In the unperturbed metric, the variance initially grows due to the enhancement of small- k modes. However, when metric perturbations are included, growth only occurs after $x \approx 10^4$ due to amplification of sub-Hubble modes. **Inset:** Evolution of the condensate, ϕ_0 , vs x , for $\xi = -70$.

In the minimally coupled multi-field case, large resonance parameters may lead to suppression of the long-wavelength modes of the preheating fields during inflation [15]. Through the constraint equation [c.f. Eq. (2.5)], this removes the resonance in the long-wavelength modes of the metric perturbations, Φ_k .

For large $|\xi|$ the same effect might be expected. In fact, for large *negative* ξ , the opposite occurs, and it is the short-wavelength modes which are suppressed relative to the $k \sim 0$ modes, leading to a red spectrum at the start of preheating [16].[†]

[†]The modes in de Sitter space evolve as Hankel functions with order $\nu = (9/4 - m^2/H^2 - 12\xi)^{1/2}$, which is positive when $\xi \ll -1$. For suppression to exist, complex ν is required.

Intuitively one sees that since the scalar curvature during inflation is $R \approx -\lambda\phi_0^2/\xi$ for $|\xi| \gg 1$ by Eqs. (2.10) and (2.9), the frequency $\omega_k^2 \equiv k^2/a^2 + 3\lambda\phi_0^2 + \xi R$ on the l.h.s. of Eq. (2.7) is $\omega_k^2 \approx k^2/a^2 + 2\lambda\phi_0^2$. This indicates that the suppression mechanism is *absent* during inflation and cannot be responsible for removing the resonances of preheating.

Before analyzing the evolution of the fluctuations during preheating numerically, we analytically estimate these quantities. Introducing new variables $\hat{\Phi}_k = \Phi_k + \delta F_k/2F$, $\hat{\Psi}_k = \Psi_k - \delta F_k/2F$, Eqs. (2.4)-(2.6) yield

$$\hat{\Phi}_k = \hat{\Psi}_k, \quad \delta\phi_k = \frac{2F^{3/2}}{\kappa^2 a \dot{\phi}_0 E} (a\sqrt{F}\hat{\Psi}_k), \quad (2.13)$$

$$\begin{aligned} & \ddot{\hat{\Psi}}_k + \left(4H + \frac{3\dot{F}}{2F}\right) \dot{\hat{\Psi}}_k \\ & + \left(\frac{k^2}{a^2} - \frac{\kappa^2 \dot{\phi}_0^2}{2F} - \frac{3\dot{F}^2}{4F^2} + \frac{\kappa^2 \lambda \phi_0^4}{4F}\right) \hat{\Psi}_k \\ & = \frac{3}{4} \left(\frac{2\ddot{F}}{F} + 3H\frac{\dot{F}}{F} - \frac{3\dot{F}^2}{2F^2} + \frac{2\kappa^2 \dot{\phi}_0^2}{3F}\right) \frac{\delta F_k}{F} \\ & - \kappa^2 \left[\xi R \phi_0 + \lambda \phi_0^3 + \frac{3}{2} \dot{\phi}_0 \left(H + \frac{\dot{F}}{2F}\right)\right] \frac{\delta\phi_k}{F}, \end{aligned} \quad (2.14)$$

where $E = 1 - (1 - 6\xi)\xi\kappa^2\phi_0^2$. Note that $\hat{\Phi}_k$ and $\hat{\Psi}_k$ are conformally transformed potentials which are derived in the Einstein frame [19]. The relation (2.13) clearly shows the link between inflaton fluctuations and metric perturbations. Eliminating $\delta F_k/F$ and $\delta\phi_k/F$ terms in Eq. (2.14) by using the relation (2.13), we obtain

$$\frac{d^2 u_k}{d\eta^2} + \left(k^2 - \frac{1}{z} \frac{d^2 z}{d\eta^2}\right) u_k = 0, \quad (2.15)$$

where $\eta \equiv \int a^{-1} dt$ is conformal time, and

$$u_k \equiv \frac{F^{3/2}}{\dot{\phi}_0 \sqrt{E}} \hat{\Psi}_k, \quad z \equiv \frac{(a\sqrt{F})}{a^2 \dot{\phi}_0 \sqrt{E}}. \quad (2.16)$$

In the long-wave limit, $k \rightarrow 0$, Eq. (2.15) is easily integrated to give

$$u_k = z \left(c_1 + c_2 \int \frac{d\eta}{z^2}\right), \quad (2.17)$$

where c_1 and c_2 are constants. Making use of Eqs. (2.13), (2.16), and (2.17), the long-wavelength solutions are

$$\Phi = -\left(\frac{1}{aF}\right) \left(c_1 - 2c_2 \int aF dt\right) + 2c_2, \quad (2.18)$$

$$\Psi = \frac{\dot{a}}{a^2 F} \left(c_1 - 2c_2 \int aF dt\right) + 2c_2, \quad (2.19)$$

$$\delta\phi = -\frac{\dot{\phi}_0}{aF} \left(c_1 - 2c_2 \int aF dt\right). \quad (2.20)$$

In the present model, the inflationary period ends when the value of $|\xi|\kappa^2\phi_0^2$ decreases to of order unity [8,11]. This means that $F = 1 - \xi\kappa^2\phi_0^2$ is of order unity during preheating, and does not change significantly. Since numerical calculations indicate that the scale factor roughly evolves as a power-law in the oscillating stage of inflaton, we can expect from Eqs. (2.18) and (2.19) that super-Hubble metric perturbations do not grow significantly during preheating, which restricts the enhancement of the inflaton fluctuations for small- k modes by Eq. (2.13). In fact, Eq. (2.20) implies that the difference between the minimally coupled case only appears in the F term, which approaches unity with the decrease of ϕ_0 .

This contradicts the earlier work [11] neglecting metric perturbations, which pointed out that the ξR term in the l.h.s. of Eq. (2.7) leads to the growth of low momentum modes for the strong coupling case. In the perturbed metric case, this term is counteracted by the last term in Eq. (2.7). Actually, eliminating the $\ddot{\Psi}_k$ term in the r.h.s. of Eq. (2.7) by using Eq. (2.6) gives rise to the $\xi R \delta\phi_k$ term in the r.h.s. of (2.7), which suppresses the resonance due to the curvature term.

In the next section, we numerically confirm this lack of resonance.

III. REMOVAL OF THE SUPER-HUBBLE RESONANCE

We numerically solved the perturbation equations (2.4)-(2.7) which are coupled to the background equations (2.8)-(2.10) through the fluctuation integrals (2.11) which mediate backreaction effects. We started integrating at the onset of preheating, with initial values of the inflaton oscillations given by

$$\phi_0(0) = \left[\frac{\sqrt{(1-24\xi)(1-8\xi)} - 1}{16\pi(1-6\xi)|\xi|}\right]^{1/2} m_{\text{pl}}, \quad (3.1)$$

which is determined by the end of slow-roll inflation [11]. We choose the conformal vacuum state for the initial field fluctuations: $\delta\phi_k = 1/\sqrt{2\omega_k(0)}$ and $\dot{\delta\phi}_k = [-i\omega_k(0) - H(0)]\delta\phi_k$. The gauge-invariant potentials Φ_k and Ψ_k are then determined when the evolution of the scalar field is known (see Eq. (34) in ref. [13]).

In the minimally coupled case, $\xi = 0$, and neglecting metric perturbations, there is a resonance band in the narrow, sub-Hubble, range [17]:

$$3/2 < \bar{k}^2 < \sqrt{3}, \quad (3.2)$$

with $\bar{k}^2 \equiv k^2/(\lambda\tilde{\phi}_0^2(0))$, where $\tilde{\phi}_0(0)$ is the initial amplitude of inflaton. In that case, the growth of field fluctuations finally stops due to the backreaction of created particles, and the final variance is $\langle\delta\phi^2\rangle \approx 10^{-7} m_{\text{pl}}^2$ [18,11].

In the perturbed spacetime with $\xi = 0$, although metric perturbations assist the enhancement of field fluctuations, only sub-Hubble modes of field and metric perturbations are amplified unless mode-mode coupling is

taken into account as in ref. [20]. We find numerically that the final variance is almost the same both in the perturbed and unperturbed metric. We warn that this is not a generic feature of preheating since in the multi-field case, alternative channels exist for resonant decay [4,5].

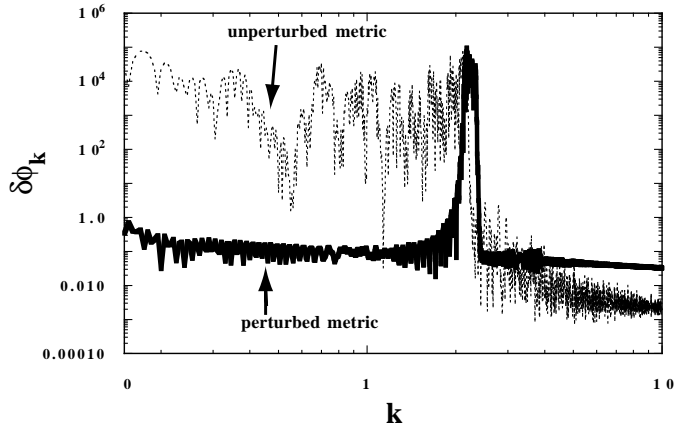


FIG. 2: The spectrum of field fluctuations: $\delta\phi_k$ vs k , for $\xi = -100$ for both the perturbed and unperturbed metrics. When metric perturbations are included, the super-Hubble resonance disappears and is replaced by a single, sub-Hubble band.

In the non-minimally coupled case with metric perturbations neglected (the *rigid* case), low momentum ($k \sim 0$) field modes undergo resonant amplification during preheating for $\xi \lesssim -10$ [11,12]. With increasing $|\xi|$, the duration during which these low momentum modes are amplified becomes gradually longer, and they dominate the final variance.

When the spacetime metric is perturbed, as it must be for consistency, these metric fluctuations on the r.h.s of Eq. (2.7) lead to a very different picture. In Fig. (1) we show the evolution of the variance $\langle\delta\phi^2\rangle$ for $\xi = -70$, both in the unperturbed and perturbed spacetimes. With metric perturbations ignored, small- k field fluctuations exhibit strong growth and the variance grows for $x \lesssim 6000$, where $x \equiv \sqrt{\lambda}\eta m_{\text{pl}}$ is the natural dimensionless conformal time of the system. For $x \gtrsim 6000$ the effect of the non-minimal coupling becomes insignificant due to the decrease in the scalar curvature. Sub-Hubble modes then dominate the variance, as in the minimally coupled case, which continues to grow until $x \sim 2.5 \times 10^4$.

When metric perturbations are included, the variance $\langle\delta\phi^2\rangle$ does not grow initially, but oscillates with much higher frequency. This implies that the small- k modes are not amplified, as expected from our earlier, analytical, discussion.

The absence of the super-Hubble resonance can be seen in Fig. (2) where we show the spectrum of $\delta\phi_k$ at constant time. In the perturbed metric, there is only a small resonance band around $k/(\sqrt{\lambda}\phi_0(0)) \simeq 2.2$, while

in the unperturbed metric, the negative coupling instability creates a resonant band that contains all modes with $k/(\sqrt{\lambda}\phi_0(0)) \lesssim 2$.

The effect of this change to the resonance structure is clearly visible in Fig. (3) where we plot a cosmological mode ($k/(\sqrt{\lambda}\phi_0(0)) = 10^{-25}$) of the metric perturbation Ψ_k , which shows no growth, and a sub-Hubble mode, which grows resonantly until backreaction ends its amplification.

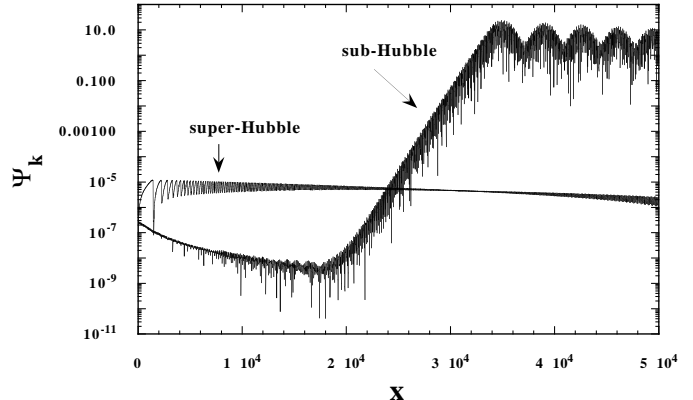


FIG. 3: The evolution of the metric fluctuation Ψ_k for the super-Hubble mode $k/(\sqrt{\lambda}\phi_0(0)) = 10^{-25}$ and the sub-Hubble mode $k/(\sqrt{\lambda}\phi_0(0)) = 2.2$ with $\xi = -100$. The evolution of Φ_k shows a similar behavior. We find that super-Hubble metric and field fluctuations do not grow during preheating in this model.

We found numerically that the final variances of field fluctuations when Φ_k is included are typically smaller, by one or two magnitudes, than in the rigid spacetime for $|\xi| \gtrsim 100$, although they are almost the same for $|\xi| \lesssim 100$. This is understandable, since field fluctuations without Φ_k for $|\xi| \gtrsim 100$ are dominated by low momentum modes and $\langle\delta\phi^2\rangle$ grows to of order ϕ_0^2 [11], while in the perturbed spacetime, resonance stops before $\langle\delta\phi^2\rangle$ reaches ϕ_0^2 , as in the case of $\xi = 0$.

The lack of strong metric preheating on cosmological scales persists in the strong coupling regime $|\xi| \gtrsim 10^3$ considered by Fakir and Unruh. The predictions of large scale metric perturbations produced in the inflationary epoch [21,22] are therefore not modified in the present model, as long as another scalar field coupled non-gravitationally to the inflaton is not introduced.

IV. DISCUSSION AND CONCLUSIONS

We have studied preheating in a non-minimal, chaotic, inflation scenario. Our main result is that consistent inclusion of metric perturbations is crucial and has a powerful and unexpected impact on the evolution of long-wavelength fluctuations during preheating, removing the

exponential growth of these modes that one finds in the absence of metric perturbations.

This is in strong contrast to the multi-field case where the negative specific heat of gravity tends to enhance any resonances that exist in their absence [4,5]. In addition, large negative ξ causes anti-suppression of long-wavelength modes, relative to short wavelength modes during inflation, providing an alternative way of avoiding the suppression mechanisms proposed in [15].

The removal of the resonances is intimately related to the existence of gauge-invariant conserved quantities in the long-wavelength limit:

$$\zeta \equiv \Psi - \frac{\dot{a}^2}{a^2 F[\dot{a}/(aF)]} \left(\Psi + \frac{a}{\dot{a}} \dot{\Psi} \right) = 2c_2, \quad (4.1)$$

which is obtained from Eq. (2.19).

A simple way to appreciate the lack of super-Hubble resonances is to recognize that the present system is conformally equivalent to a minimally coupled, single scalar field, model with potential $\hat{V}(\phi) \equiv V(\phi)/(1 - \xi\kappa^2\phi^2)^2$ and $|\xi|\kappa^2\phi^2 \sim 1$ at the end of inflation. Hence it is not surprising that the resonance structure is similar to that of the minimally coupled case [17]. However, this equivalence is missing if metric perturbations are not included.

On the other hand, on sub-Hubble scales, we find that both field and metric fluctuations are excited during preheating as in the minimally coupled case. This may result in an important cosmological consequences such as the production of primordial black holes [4]. There may also be interesting implications for non-thermal symmetry restoration due to the change in the time evolution of the variance.

Although we have restricted ourselves to a non-minimally coupled scalar field in Einstein gravity, the evolution of metric perturbations can be analyzed in a unified manner in generalized Einstein theories, which include the higher-curvature, Brans-Dicke, and induced gravity theories [13]. As long as the system we consider is a single-field model, and stress-energy is conserved, super-Hubble metric perturbations will not be enhanced during preheating, because conserved quantities, which generalize ζ in Eq. (4.1), exist.

However, introducing a coupling such as the standard $g^2\phi^2\chi^2/2$, will violate the conservation of the quantity ζ for certain values of g and ξ , as occurs in the minimally coupled multi-field case [5].

Finally it is of great interest to examine the realistic multi-field case in known classes of inflationary models, because we can constrain the inflaton potential in terms of the distortion of the CMB spectrum due to parametric resonance in preheating. This requires a complete study of backreaction including metric perturbations, which we leave to future work.

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